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MODELING OF THE MECHANISM OF DECREASING THE HYDRODYNAMIC DRAG OF BODIES BY THE SURFACE RUNNING WAVE METHOD

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We explore the possibility of radically decreasing the hydrodynamic drag by changing the external flow so that the boundary layer is replaced by some flow with a small velocity gradient.

The problem of decreasing the hydrodynamic drag has always been of great theoretical and practical interest. Extensive studies in this field deal with a common problem of controlling the boundary layer based on the Prandtl model. With such a formulation of the problem the possibilities of decreasing drag by annihilating or radically changing the boundary layer could not be considered. The principal feature of the boundary layer is that the velocity gradient in it increases simultaneously with increasing Reynolds number. This explains the nonproportionally slow decrease in the drag coefficient with increasing Reynolds number. The traditional uncontrolled flow does not permit the drag to be considerably decreased. And it is only the use of some control means, for example, a boundary deformed by the running-wave law, that enables one to get rid of the boundary layer in both the physics of the flow and its mathematical model. This was theoretically substantiated by V. I. Merkulov [1]. The point is that with a certain choice of parameters the running surface wave will generate a Tollmien–Schlichting wave in the flow, but with one difference: the stationary surface always does negative work on the flow, which leads to an instability of the Tollmien–Schlichting wave and a flow turbulization, whereas the surface wave is able to do both positive and negative work on the flow and thus control the flow as to both the wave length and the phase velocity. This means that under the action of a finite-amplitude surface wave Tollmien–Schlichting waves can evolve into some quasi-periodic flow characterized by a small gradient and small viscous losses. With a proper choice of the phase velocity of the running wave it is possible to provide, in the front part, an energy flow from the liquid to the elastic coat and thus save the flow energy in the form of energy of elastic vibrations. In turn, in the stern part it is necessary to restore the pulse in the liquid and decrease the energy of elastic vibrations.

The above considerations guarantee an order-of-magnitude decrease in the drag, but they are theoretical and their experimental check in a wide range of diagnostic variables is a complicated and expensive project. Mathematical modeling of the above-described approach to the drag decrease will permit a radically more effective verification of the above conception (although one cannot manage without subsequent experimental testing of results obtained in modeling, but this is already in the scope of the possible effect). In so doing, it is necessary to get answers to the following basic questions:

a) can the wave running on the surface of a body really form a periodic flow;

b) what is the qualitative and quantitative influence of the wave parameters (amplitude, phase velocity, frequency) on the flow control;

c) what are the laws of change in the phase velocity and amplitude along the body minimizing the hydrodynamic drag.

Some of the results containing answers to these questions are presented in [2].

In the present paper, the viscous fluid flow along a boundary deformed according to the given law is investigated. It should be noted that the law of deformation (amplitude, phase velocity) and the longitudinal velocity gradient are varied over such ranges that will make it possible in the future to use the results obtained to analyze an arbitrary model or construct a specific model. The fluid flow is modeled by solving complete Navier–Stokes equations.

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Formulation of the Problem. Consider a viscous fluid flow in the upper semiplane with a unit velocity oriented along the x-axis to the right. The periodic solution we are interested in is only possible under certain, also periodic, boundary conditions. In investigating the running-wave mechanism, we shall seek the boundary conditions at the semispace boundary $y = 0$ in the form

$$u = 0\ , \quad v = aU \sin(\omega t - kx)\ . \tag{1}$$

In a coordinate system moving to the right at a velocity $U_{ph} = \omega/k$, we obtain the following stationary conditions at the boundary $y = 0$:

$$u = -U_{ph}\ ; \quad v = aU \sin kx$$

and

$$u = 1 - U_{ph} \tag{2}$$

— at infinity.

For the stationary boundary conditions, the problem on the fluid motion in the upper semiplane can have a stationary solution, which is expressed in terms of the current function.

From the boundary conditions (2) for the values of the current function and its derivatives we obtain

$$-\psi(x, 0) = \frac{aU}{k} \cos kx + \text{const}\ , \quad \frac{\partial \psi(x, 0)}{\partial y} = -U_{ph}\ . \tag{3}$$

The boundary conditions permit constructing an approximate value of the current function in the vicinity of the boundary:

$$\psi(x, y) = \frac{aU}{k} \cos(kx) - yU_{ph} + \text{const}\ .$$

Hence we can find the $y(x)$ curve on which the current function has a constant value:

$$y = A \cos(kx) + \text{const}\ .$$

This line can be chosen as a boundary each point of which is moving tangentially towards the flow at the phase velocity. Such boundary conditions can be realized physically at an elastic boundary if transverse vibrations by the running-wave law are provided on it. The consideration of the flow in the region where $-\infty < x < \infty$ excluded the evolution of the process characteristic of the flow past a finite body.

Calculation Region. The low viscosity, which in dimensionless variables corresponds to a large Reynolds number, permits separating an external potential flow and a narrow near-boundary region of a gradient flow, whose thickness has a value of the order of $1/\sqrt{\text{Re}}$. Let us introduce the concept of a geometric boundary layer, which also represents a narrow region adjoining the boundary, whose thickness is independent of the Reynolds number and is determined by the condition $\delta_g << 1$. It is in this region that numerical modeling of the flow is realized.

Differential Equations. Taking into account the smallness of the region under consideration and the sufficient smoothness of the body surface, we can write the Navier–Stokes equations in the form

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\text{Re}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \quad \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\text{Re}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\ . \tag{4}$$

Unlike the dynamic boundary-layer model, we cannot demand fulfillment of the conditions

$$\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}\ , \quad \frac{\partial p}{\partial y} << 1\ ,$$

TABLE 1. Test Calculation of the Wedge Flow

ξ	u_{ss}	u_{cal}	ε	ξ	u_{ss}	u_{cal}	ε
0.1	0.0903	0.0499	0.0404	0.8	0.5834	0.600	0.0166
0.2	0.1756	0.151	0.0246	0.9	0.6344	0.651	0.0166
0.3	0.2558	0.270	0.0142	1.0	0.6811	0.716	0.0349
0.4	0.3311	0.342	0.0109	1.2	0.7615	0.776	0.0145
0.5	0.4015	0.424	0.0225	1.4	0.8258	0.840	0.0142
0.6	0.4670	0.474	0.0070	1.6	0.8860	0.914	0.0280
0.7	0.5276	0.540	0.0124	1.8	0.9141	0.971	0.0569

that permit making a transition to parabolic Prandtl equations. This is explained by the fact that the running wave should transform the boundary layer into a periodic flow in which these conditions are not met.

Boundary Conditions. The region boundary in the chosen coordinate system is written as follows:

$$y_0(x, t) = A(x) \sin(\omega t - kx) .$$

The wave number and the amplitude are a function of x. At the region boundary the fluid motion coincides with the motion of the boundary points:

$$u(x, y_0) = 0 , \quad v(x, y_0) = \frac{dy_0}{dt} = A\omega \cos(\omega t - kx) .$$

The external flow velocity U was taken in the form of a power function of x: $U(x) = cx^m$. Such a function corresponds to the flow past conical bodies. The parameter m is related to the cone angle α by the following formula [3]:

$$\alpha = \frac{6m\pi}{3m + 1} .$$

The angle $\alpha = \pi$, which is attained at $m = 1/3$, corresponds to the flow near the front critical point. The value of $\alpha = 0$ corresponds to the flow along the cylindrical part of the body and $\alpha < 0$ — to the flow in the stern part of the body.

Normalization of the dimensional values was carried out in the following way. As a length unit, we took the length of the calculation region and as a velocity unit — its maximum value, which was attained at the right end of the calculation region for the "bow" part of the body and at the left end of the region — for the "stern" part. The density is constant and equal to unity and the pressure is normalized to the kinetic head.

In our calculations, the region boundary was deformed by the law of the running wave with a variable amplitude. The wave length changed in each portion of the flow region, since it was determined by the value of the phase velocity. The latter was chosen in proportion to the flow velocity and changed together with it.

By the results of the solution the work of the total pressure forces

$$F_n = \int \left(p + \frac{\rho u^2}{2} \right) u_n ds$$

on normal boundary displacements and the friction force F_τ in each portion were determined. In dimensionless variables, the quantity of this work coincides with the power factor, which for the work of the friction forces coincides with the friction factor.

Algorithm. Calculations were carried out by the large-particle method [4] relying on the splitting of the input nonstationary system of Navier–Stokes equations written in the form of conservation laws according to the physical processes. The stationary solution of the problem, if it exists, is sought in the process of multiple fulfillment of time steps.

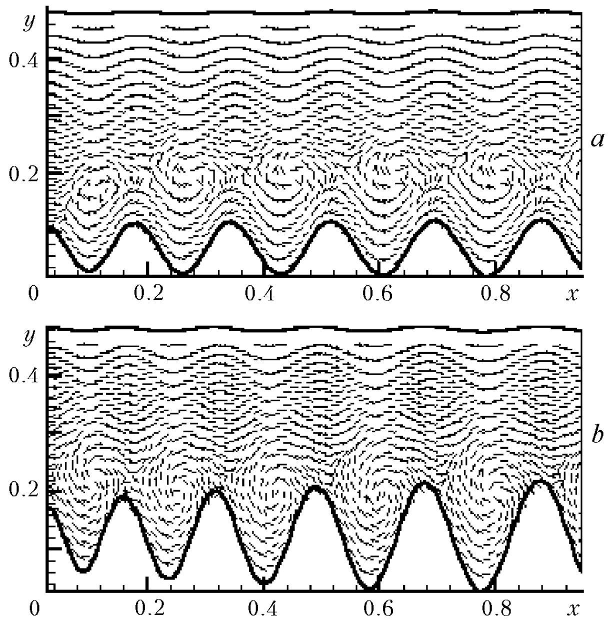

Fig. 1. Velocity field in the vicinity of the boundary deformed by the running-wave law with A = 0.1 (a) and A = 0.2 (b).

Test Calculation. To get an idea of the accuracy of the algorithm used, we calculated the velocity profile on a wedge for which the solution is known [3]. Assuming $3m = m'$, we will be able to apply all of the results obtained to a plane problem of the flow past wedges with various angles.

Table 1 gives the data of the test calculations of the wedge flow. The first column contains the values of the nodes of the variable ξ related to the distance y from the boundary by the relation

$$\xi = \sqrt{\frac{m+1}{2}\frac{c}{\mu}}\, y x^{\frac{m-1}{2}} ,$$

where c = 1, m = 1/3, μ = 0.01, and x = 0.4. The second column contains the velocity values obtained from the self-similar solution from the viewpoint of the boundary-layer model. The third column gives the velocity values in these nodes obtained in the calculations, and the fourth one contains the absolute error values for the given velocity profile. The maximum error is 6%.

Results of the Calculations. Figure 1a shows the velocity field of the cone flow with parameters U_{ph}/U = 0.5, m = 0.2, A = 0.1, and Re = 100. For illustration of the flow pattern, the vertical coordinate is magnified five times. For better visualization of the velocity field in the vicinity of the running wave, the visualization grid was chosen to be equidistant to the running wave. This led to the fact that the crowding of arrows away from the running wave does not coincide with the trajectory. At the same time, the arrow length and orientation completely correspond to the value and orientation of the velocity vector at the points of arrow positions. In Table 2, this regime corresponds to the first row. The subcolumn of five values in the second column corresponds to the five waves situated in the calculation region. As is seen from Table 2, the friction factor is within the 0.0028–0.0110 range, whereas in the boundary layer on a smooth plate the friction factor determined by the formula $0.66/\sqrt{\text{Re}}$ is equal to 0.066 [3]. Thus, the friction force in the boundary layer at Re = 100 is an order of magnitude larger than on the running wave. The latter has changed the flow so that the velocity gradient at the boundary has decreased by an order of magnitude.

To elucidate the dependence of the friction factor on the Reynolds number, compare this flow to flow No. 2 (see Table 2), which differs only in the fluid viscosity. It is seen that the friction coefficient differs by an order of magnitude and for the flow with a running wave decreases in inverse proportion to the first degree from the Reynolds number. For the boundary layer, a tenfold increase in the Reynolds number will be followed by only a threefold decrease in the friction factor. The revealed inversely proportional dependence of the friction coefficient on the Reynolds

TABLE 2. Work of Total Pressure Forces and Friction Forces for Various Flowing Conditions

Calculation variant number	F_n	F_τ	U_{ph}/U	m	A	Re
1	-0.097 -0.026 -0.026 -0.020 -0.012	-0.00286 -0.00555 -0.00775 -0.00924 -0.01108	0.5	0.1	0.1	100
2	-0.042 -0.027 -0.020 -0.017 -0.014	-0.00056 -0.00055 -0.00055 -0.00054 -0.00058	0.5	0.1	0.1	1000
3	-0.009 -0.003 -0.004 -0.003 -0.003	-0.00110 -0.00094 -0.00091 -0.00098 -0.00094	0.5	0.0	0.1	1000
4	0.062 0.023 0.012 0.003	-0.00097 -0.00036 -0.00037 -0.00041	1.2	-0.1	0.1	1000
5	-0.342 -0.143 -0.101 -0.086 -0.073	-0.00005 -0.00012 -0.00013 -0.00019 -0.00018	0.5	0.2	0.2	1000

number permits determination of the friction factor for any value of the laminar Reynolds number, since in the case under consideration the Reynolds number is determined by the wave length rather than by the body size.

To elucidate the role of the running-wave amplitude, consider two flows — flows 2 and 5 (see Table 2), one of which is created by the running wave with an amplitude of 0.1 and and other — with an amplitude of 0.2. Figure 1b shows the velocity field corresponding to flow 5.

If, for comparison, we use the values of the central wave, then it may be concluded that the friction drag in the second case turns out to be twice as small and the work of the pressure forces — twice as large. The negative value of the work corresponds to the energy flow from the side of the fluid flow to the elastic boundary, which can be used to excite the running wave by increasing the pressure drag. Unlike the friction forces, whose work will inevitably transform to heat, this part is determined by the damping of elastic vibrations in the material. If the material features a fairly small damping, then the waves excited in the bow part will propagate downstream on the cylindrical part of the body and reach the stern conical part. In Table 2, row 3 corresponds to the middle cylindrical part of the body and row 4 — to the stern part.

The work of the friction forces in the presence of a running wave appears to be very small. The basic energy losses and gains will be associated with the work of the pressure forces. As follows from Table 2, in the bow part it has a negative sign, which causes a stagnation of the flow and creates a drag. In the cylindrical part of the body the work of the pressure forces is negligibly small. In the stern part at the chosen phase velocity of 1.2 it has a positive value and, in total, exceeds the negative work in the first two parts. Such work leads to an increase in the running-wave amplitude and creates a draft on the housing. The calculations confirm the theoretically revealed (in [1]) possibility of returning the energy of elastic vibrations generated in the bow part at the cost of the flow energy to the stern part.

The above examples of constructing a flow consisting of three parts illustrate only some of the model situations pointing to the potentialities of this method.

CONCLUSIONS

1. The running wave considerably decreases the friction drag, a natural result of which will be a flow without separation with a deep bow pressure recovery and elimination of the form drag.

2. Compared to the flow around a solid body, the running wave adds the negative (breaking) work of the pressure forces in the bow part of the body and the positive (accelerating) work of these forces in the stern part of the body. With a good design of the elastic coat, these works will compensate one another and the power required for motion will be determined by the dissipative losses in the coat material.

3. To excite a running wave in the stern part, the phase velocity should be lower than the flow velocity. In the calculation examples, it had a value of $0.5U$. In the middle part, the phase velocity should be equal to the flow velocity. To create a pushing force in the stern at the cost of the vibrational energy of the elastic coat, the phase velocity should be higher than the flow velocity. In the calculation examples, it has a value of $U_{ph} = 1.2U$.

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NOTATION

a, arbitrary parameter; A, relative amplitude of the running wave; c, constant at a power representation of the velocity at the outer boundary of the wall boundary layer; ds, boundary surface element; F_n, dimensionless work of total pressure forces on normal displacements of the boundary; F_τ, dimensionless work of friction forces on horizontal displacements of the boundary; k, wave number; m, exponent of a conical flow; m', exponent for the wedge flow velocity; p, pressure; Re, Reynolds number; t, time; u, xth component of the velocity vector; U, external flow velocity; u_a, values for the velocity profile obtained from the self-similar solution; U_{ph}, phase velocity of running-wave propagation; u_{cal}, calculated values for the velocity profile at a wedge flow; u_n, normal component of the boundary velocity; v, yth component of the velocity vector; x, y, Cartesian coordinates; $y_0(x, t)$, movable region boundary; α, cone angle; δ_g, geometric boundary-layer thickness; ε, local absolute error; μ, first viscosity; ρ, density; ω, circular wave frequency; ξ, dimensionless ordinate of the laminar boundary layer; $\psi(x, y)$, current function. Subscripts: ss, self-similar; g, geometric; cal, calculated; ph, phase; n, normal to the boundary surface; τ, tangent to the boundary surface; 0, lower ($y = 0$) region boundary.

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